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BT-MRE-.I&II(S)-05.14-0436

B.Tech. Degree I and II Semester Supplementary Examination in Marine Engineering May 2014

MRE 101 ENGINEERING MATHEMATICS I

Time : 3 Hours Maximum Marks : 100

I. (a) Evaluate $\underset{x\to 0}{Lt}\left(\frac{1}{Sinx}-\frac{1}{x}\right)$ (6)

(b) Evaluate $\underset{x\to 0}{Lt}\ \frac{xe^x-\log(1+x)}{x^2}$ (7)

(c) Verify Rolle's Theorem for $\frac{Sin\,x}{e^x}$ in $(0,\pi)$ (7)

OR

II. (a) Find the n[th] derivative of $x^2 \log 3x$. (5)

(b) Show that the radius of curvature at any point of the cycloid $x=a(\theta+Sin\theta)$; $y=a(1-Cos\theta)$ is $4a\,Cos\,\theta/2$ (6)

(c) $If\ y=\left(Sin^{-1}x\right)^2$, show that $\left(1-x^2\right)y_{n+2}-(2n+1)x\,y_{n+1}-n^2y_n=0$. (9)

III. (a) Given $u=Sin\left(x/y\right)$; $x=e^t$; $y=t^2$ find $\frac{du}{dt}$ as a function of t. (6)

(b) Find the plane curve of fixed perimeter and maximum area. (6)

(c) Discuss the maxima and minima of $f(x,y)=x^3y^2(1-x-y)$ (8)

OR

IV. (a) $If\ u=\frac{x+y}{1-xy}$ and $V=Tan^{-1}x+Tan^{-1}y$ find $\frac{\partial(u,v)}{\partial(x,y)}$ (8)

(b) $If\ u=Sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=Tan\,u$ (6)

(c) The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm and 6cm respectively. The possible error in each measurement is 0.1cm. Find approximately the maximum possible error in the values computed for the volume and the lateral surface. (6)

V. (a) Show that the equation $y^2-x+4y+5=0$ represents a parabola. Find its axis, vertex, focus and directrix. (8)

(b) Find the asymptotes of the hyperbola $3x^2-5xy-2y^2+17x+y+14=0$. (6)

(c) Find the condition the straight line lx+my+n=0 may touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (6)

OR

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VI.	(a)	If e and e' are eccentricities of a hyperbola and its conjugate show that $\frac{1}{e^2}+\frac{1}{e'^2}=1$.	(6)
	(b)	Show that the locus of the point of intersection of perpendicular tangents to the parabola $y^2=4ax$ is the directirix $x+a=0$.	(6)
	(c)	Find the focii, centre, equations of the directices, length of major and minor axes of the ellipse $9x^2+4y^2+36x-8y+4=0$	(8)
VII.	(a)	Find a Reduction formula for $\int Sin^n x dx$. Hence evaluate $\int_0^{\pi} Sin^5 x\, dx$.	(10)
	(b)	Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$.	(10)

OR

VIII.	(a)	Find the surface of the solid formed by revolving the cardioid $r = a\,(1+Cos\theta)$ about the initial line.	(7)
	(b)	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$.	(7)
	(c)	Find the length of one arc of the cycloid $x=a(t-S\text{int})$; $y=a(1-Cost)$	(6)
IX.	(a)	If G is the centroid of a triangle ABC prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=0$	(6)
	(b)	Prove that $\left[\vec{B}\times\vec{C}\quad \vec{C}\times\vec{A}\quad \vec{A}\times\vec{B}\right]=\left[\vec{A}\ \vec{B}\ \vec{C}\right]^2$.	(7)
	(c)	Show that the points $3\bar{i}-2\bar{j}+4\bar{k}$, $-6\bar{i}+3\bar{j}+2\bar{k}$, $13\bar{i}+17\bar{j}-\bar{k}$, $5\bar{i}+7\bar{i}+3\bar{k}$ are coplanar.	(7)

OR

X.	(a)	Find divergence and curl of $\vec{F}=3x^2\bar{i}+2xy^3\bar{j}+4xz^2$ at (1, 2, -1).	(5)
	(b)	A vector field $\vec{F}$ is of the form $\vec{F}=(6xy+z^3)\bar{i}+(3x^2-z)\bar{j}+(3xz^2-y)\bar{K}$. Show that $\vec{F}$ is a conservative field and scalar potential.	(7)
	(c)	Prove that $\nabla^2(r^n)=n(n+1)r^{n-2}$.	(8)

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